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EFFECTIVE FIELD THEORY APPROACH TO COSMOLOGICAL INITIAL CONDITIONS: SELF-CONSISTENCY BOUNDS AND NON-GAUSSIANITIES

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Abstract

Effective Field Theory (EFT) is an efficient method for parametrizing unknown high energy physics effects on low energy data. When applied to time-dependent backgrounds, EFT must be supplemented with initial conditions. In these proceedings, I briefly describe such approach, especially in the case of inflationary, almost-de Sitter backgrounds. I present certain self-consistency constraints that bound the size of possible deviations of the initial state from the standard thermal vacuum. I also estimate the maximum size of non-Gaussianities due to a non-thermal initial state which is compatible with all bounds. These non-Gaussianities can be much larger than those due to nonlinearities in the action describing single-scalar slow roll inflation.

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1 EFT Approach to the Choice of Initial Conditions

1.1 Introduction

The possibility of observing very high-energy, “trans-Planckian” physics in the cosmic microwave background radiation, thanks to the enormous stretch in proper distance due to inflation, is one of the most exciting possibility for probing string theory, or any other model of quantum gravity. As such, it has received considerable attention, once the possibility was raised that these effects could be as large as H/M, with H the Hubble parameter during inflation, and M the scale of new physics (e.g. the string scale). A partial list of references is given in [1], on which this contribution is largely based.

Due to our ignorance of the ultimate theory governing high-energy physics, the most natural, model-independent approach to studying modifications to the primordial power spectrum is effective field theory (EFT) [2, 3]. Using an EFT approach, the authors of [2] concluded that the signature of any trans-Planckian modification of the standard inflationary power spectrum is $O(H^2/M^2)$, well beyond the reach of observation even in the most favorable scenario ($H \sim 10^{14}\,\mathrm{GeV}, M \sim 10^{16}\,\mathrm{Gev}$).

What was absent from e.g. ref. [2] was a *systematic* EFT approach to initial conditions. That work presented convincing arguments against the (in)famous α-vacua [4] of de Sitter space, but it did not give a complete parametrization of finite-energy non-thermal states.

That parametrization was given in [5], where the EFT approach was systematically extended to the choice of initial conditions.

1.2 The “Initial” State

Let me review a suitably modified version of the approach of [5].

The most important difference between [5] and other approaches is that in [5] initial conditions for modes of all wavelengths are specified at the same

initial time t^*. Other approaches give the initial conditions separately for each mode, at the time it crosses the horizon. The latter prescription is useful in the context of inflationary cosmology, but it obscures the field-theoretical meaning of the perturbation and/or initial condition: it does not easily account the fact that after t^* curvatures and energy densities are small, so the field theory is under control, and it does not easily translate into an EFT language. The former prescription, instead, leads naturally to a simple classification of initial conditions in terms of *local* operators defined at the space-like boundary (i.e. initial surface) $t = t^*$.

The prescription start by supplementing the EFT action describing all relevant low energy fields with a boundary term that encodes the standard thermal vacuum. To be concrete, we will work out the example of a massless scalar field in a time-dependent background. The 4d (bulk) action plus a 3d boundary term is

$$\begin{aligned} S &= S_4 + S_3, \qquad S_4 = \int_{t \geq t^*} d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi \\ S_3 &= \int_{t=t^*} d^3x \sqrt{\gamma(x)} \int_{t=t^*} d^3y \sqrt{\gamma(y)} \phi^*(x) \kappa(x,y) \phi(y). \end{aligned} \tag{1}$$

Here γ_{ij} is the induced metric on the surface $t = t^*$. The role of S_3 is to specify the wave functional for the scalar ϕ at $t = t^*$:

$$\Psi[\phi(x)] = \exp(iS_3[\phi]). \tag{2}$$

Selecting an initial state for ϕ means in this language to choose a particular $\kappa(x, y)$. For instance, in de Sitter space with line element

$$ds^2 = a(\eta)^2(-d\eta^2 + dx^i dx^i), \quad a(\eta) = -\frac{1}{H\eta}, \quad i = 1, 2, 3, \quad -\infty < \eta < 0, \tag{3}$$

the standard thermal [6] vacuum is obtained by choosing

$$\tilde{\kappa}(k) = -\frac{k^2\eta^*}{1 - ik\eta^*}. \tag{4}$$

Here, a tilde denotes the Fourier transform from space coordinates to co-moving momenta k^i ($k \equiv \sqrt{k^i k^i}$) and η^* is the initial (conformal) time.[1] This expression for κ makes clear that the choice of such initial time is conventional, since a change in η^* changes only κ, not the wave functional. From now on the standard vacuum functional will be called $|0\rangle$.

1.3 Changing the Initial State

Next, we want to find a convenient classification of changes in the initial state. This can be done by adding a new boundary term to the action: $S \to S + \Delta S_3$. To determine ΔS_3, we notice that, at any finite time t after t^*, we are insensitive to changes that only affect very low co-moving momenta k: co-moving momenta $k < H(t)a(t)$ correspond to perturbations with super-horizon physical wavelength $\lambda_p > 1/H(t)$, which are unobservable at time t. So, since we are interested in changes that can be observed in the CMB of the present epoch, we have an IR cutoff naturally built into the theory. This IR cutoff tells us that observable changes in the initial conditions can be parametrized by *local* operators:

$$\Delta S_3 = \sum_i \beta_i M^{3-\Delta_i} \int_{t=t^*} d^3x \sqrt{\gamma} O^i. \tag{5}$$

Here O^i are operators of scaling dimension Δ_i, M is the high-energy cutoff of the EFT and the β_i's are dimensionless parameters. The dimension Δ_i determines among other things how "blue" is the change in the power spectrum: the fractional change in the power spectrum is proportional to $k^{\Delta_i - 2}$. Since the EFT makes sense only for $k < M$, operators of high conformal dimension do not significantly change the observable spectrum. So, the most significant observable changes in the primordial fluctuation spectrum are parametrized by a few local operators of low conformal dimension.

[1] From now on, η, η^* will denote the conformal time, t, t^* will denote the synchronous proper time, and $a()$ will always denote the scale factor.

We just mentioned that the EFT needs a UV cutoff. This means that the operators O_i have to be suitably regulated at short distance. In other words, they are local only up to the cutoff scale M. As a simple example, consider the dimension-four operator $O^4 = (\beta/M)(\partial_i\phi)^2$. It has to be smeared at short distance, for instance by the replacement

$$\partial_i\phi\partial_i\phi \to \partial_i\phi f(-\partial^2/a^2(t^*)M^2)\partial_i\phi. \tag{6}$$

Here $f(x)$ is a smooth function obeying $f(x) = 1$, for $x \leq 1-\epsilon$; $f(x) = 0$, for $x \geq 1+\epsilon$; ϵ is a small positive number. The scale factor $a(t^*)$ appears because we want to cutoff at M the *physical* momentum $k/a(t^*)$, not the co-moving momentum k.

1.4 Power Spectrum

As a first application, let us derive the change in the power spectrum of a minimally-coupled scalar field in de Sitter space, induced by the operator O^4 [5]. The change in initial conditions ΔS_3, is equivalent to perturbing the Hamiltonian of the system by an instantaneous interaction $H_I = -\delta(t - t^*)\Delta S_3$. So, the perturbed power spectrum is

$$P(k) = \lim_{\eta\to 0^-}\langle|\phi(k,\eta)|^2\rangle = \lim_{\eta\to 0^-}\langle 0|\exp(-i\Delta S_3)|\phi(k,\eta)|^2\exp(i\Delta S_3)|0\rangle. \tag{7}$$

To first order in β, the change is

$$\delta P(k) = -i\frac{\beta}{M}\int d^3x\langle 0|[O^4(x), |\phi(k,0)|^2]|0\rangle. \tag{8}$$

This quantity is easily computed in terms of commutators of free fields in de Sitter space, resulting in [5, 1]

$$\delta P(k) = -\frac{\beta}{M}\frac{H^2}{k^3}\mathrm{Im}\,[\phi^+(\eta^*,k)]^2k^2f(k^2/a^2(\eta^*)M^2). \tag{9}$$

H^2/k^3 is the unperturbed power spectrum, while the canonically normalized, positive frequency solution of the free-field equations of motion is

$$\phi^+(\eta,k) = \frac{H}{\sqrt{2k^3}}(1-ik\eta)\exp(ik\eta). \tag{10}$$

For $k\eta^* \sim 1$, the effect of O^4 on the power spectrum can be as large as $\delta P/P \sim \beta H/M$, i.e. in the observable range when β is $O(1)$.

2 Back-Reaction and Calculability Bounds

Reference [5] does not take into account all effects due to the back-reaction of the modified stress-energy tensor on the metric. Specifically, any change in the boundary conditions of the effective field theory generates modifications to the expectation value of the stress-energy tensor. These modifications can become large near the (space-like) boundary hypersurface. By requiring that the back-reaction remains under control, we shall get new bounds on the size of the parameters β_i.

From now on, unless otherwise stated, we will set $a(t^*) = 1$ for ease of notation.

Since gravity couples universally to matter through the stress-energy tensor, any change in the expectation value of T_μ^ν will back-react on the metric and change the background, that will be no longer a pure de Sitter space. In our formalism, the change in $\langle T_\mu^\nu \rangle$ to first order in the β_i's is easily written as

$$\delta\langle T_\mu^\nu(t,x)\rangle = -i\langle 0|[\Delta S_3, T_\mu^\nu(t,x)]|0\rangle. \tag{11}$$

Notice that we are looking for a first-order change in $\langle T_\mu^\nu \rangle$. This is different from *second-order* effects due to the change in the vacuum, considered elsewhere in the literature [7], such as particle production & c. The change we are considering here is vanishingly small for times $\Delta t = t - t^* \gg 1/M$, but it can be large for times Δt of order $1/M$. A direct computation [1] shows that under the perturbation O^4, $\langle T_0^0 \rangle$ does not change to first order in β, while $\delta\langle T_i^i \rangle$ can be as large as

$$\delta\langle T_i^i(t,x)\rangle \approx \beta M^4 g(\Delta t M). \tag{12}$$

The exact form of the function $g(x)$ depends on the shape of the cutoff

function f, but it is always $O(1)$ inside the region $x \sim 1$ and it vanishes for $x \gg 1$.

Now, a change in the pressure, $\delta p = \delta\langle T^i_i(t,x)\rangle$, implies a change in the Hubble constant:

$$\delta\dot{H} = 4\pi G\delta p \approx 4\pi G\beta M^4, \qquad H = \frac{\dot{a}}{a}\ . \tag{13}$$

Combined with the standard slow-roll conditions $\dot{H} = \epsilon H^2$, $\ddot{H} = 2\epsilon\eta' H^3$,[2] this equation, and the obvious estimate $\dot{g}(\Delta t M)|_{t\approx t^*} \sim M$, implies severe constraints on β:

$$\beta \leq \epsilon\frac{1}{4\pi G M^2}\frac{H^2}{M^2}, \qquad \beta \leq 2\epsilon\eta'\frac{1}{4\pi G M^2}\frac{H^3}{M^3}. \tag{14}$$

They can easily rule out the observability of any change in the power spectrum.

2.1 What The Bounds Mean

Since the back-reaction effect we found is limited to a short time of order $1/M$ after t^*, one may think that it should be possible to relax the slow roll conditions for such a short time without any observable consequence on the power spectrum, except for those modes that cross the horizon within a time Δt after t^*. This is not so, because the correct way of thinking about ΔS_3 is as a parametrization of all changes that happened at *any* time before t^*. In other words, all modes that cross the horizon *before* t^* can (and are) affected significantly. Consider in particular the case of single-scalar slow roll inflation. The scalar fluctuations of the metric are described by a gauge invariant variable v, whose action is that of a minimally coupled free scalar with a time-dependent mass term (see [8] and references therein)

$$\begin{aligned} S &= \int d^3x dt a^3\left[-\dot{v}^2 + a^{-2}(\partial_i v)^2 + \mathcal{M}^2 v^2\right], \\ \mathcal{M}^2 &= -3\dot{H} + \frac{3H\ddot{H}}{2\dot{H}} - \frac{2\ddot{H}}{H} + \frac{2\dot{H}^2}{H^2} - \frac{\ddot{H}^2}{4\dot{H}^2} + \frac{\dddot{H}}{2\dot{H}}\ . \end{aligned} \tag{15}$$

[2]Typically, $\epsilon, \eta' \leq 10^{-2}$.

The change in $\dot{H}$ is confined to within a time $\Delta t \sim 1/M$, so, in looking at modes of wavelength longer than the cutoff $1/M$, we can approximate its effect by replacing the time-dependent terms induced in $\mathcal{M}$ by the back-reaction with $\delta(t - t^*)\delta\mathcal{M}/M$. This is the same as adding a new operator in ΔS_3: $O = \int d^3x(\delta\mathcal{M}/M)v^2$. When $\dot{H} \gg \epsilon H^2$ the new induced boundary term $\delta\mathcal{M}/M$ is $O(M)$, and it changes the spectrum as [9]

$$\frac{\delta P(k)}{P(k)} = \frac{M}{H}, \qquad \text{for } k\eta^* \ll 1. \tag{16}$$

This change is *never* a small perturbation of the de Sitter space result, so we must satisfy the slow-roll condition $\dot{H} \ll \epsilon H^2$ and re-evaluate the change in $\delta\mathcal{M}/M$. It can be estimated as [9] $\sim \beta M^5 G/\epsilon H^2$. By asking again that the change in the power spectrum $\delta P(k)/P(k)$ is not greater than $O(1)$ we find estimate $\beta \leq O(\epsilon H^3/GM^5) \sim 10^{-2}$. If we ask that the change is smaller than the one computed at tree level [Eq. (9)] we must have $\dddot{H} \ll \epsilon\eta\eta' H^4$, where η' is another slow-roll parameter of magnitude comparable to ϵ and η. This gives a very strong estimate: $\beta \leq O(\epsilon\eta\eta' H^4/GM^6)$.

This method for arriving at a bound is more involved than that leading to Eq. (14), but it is more satisfying. The slow roll expansion is not assumed to be valid at times infinitesimally close to t^*, and the meaning of the bound is clearer: if we do not impose it, then the change in the power spectrum computed in ref. [5] and by Eq. (9) is in reality sub-dominant compared to that due to the back-reaction on the metric.

3 Non-Gaussianities

Other changes to the initial state exist, that give a potentially observable signal while being compatible with back-reaction bounds. One such change is a non-Gaussianity *in the initial conditions*. It is induced by the boundary

action[3]

$$\Delta S_3 = \int_{\eta=\eta^*} d^3 x a^3 \lambda v^3. \tag{17}$$

To first order in λ, this term induces a three-point function for v [9]:

$$\langle \tilde{v}(k_1)\tilde{v}(k_2)\tilde{v}(k_3)\rangle = -i\lambda \int d^3 x a^3 \langle 0|[v^3(\eta^*, x), \tilde{v}(0,k_1)\tilde{v}(0,k_2)\tilde{v}(0,k_3)]|0\rangle. \tag{18}$$

A short calculation gives

$$\begin{aligned}\langle \tilde{v}(k_1)\tilde{v}(k_2)\tilde{v}(k_3)\rangle &= -(2\pi)^3 \delta^3(k_1+k_2+k_3)\frac{\lambda H^3}{4}\sum_{i>j} k_i^{-3}k_j^{-3}, \quad |k_i|\eta^* \ll 1, \\ &\approx 0, \qquad |k_i|\eta^* \gg 1. \end{aligned} \tag{19}$$

This functional dependence is similar to the universal non-Gaussianities due to the bulk gravitational action [10]; except for the cutoff effect at $|k_i|\eta^* \sim 1$, which is absent in the bulk effect. This cutoff is another illustration of the fact that the boundary term ΔS_3 is physically equivalent to changing the evolution of space-time at all times before η^*: wavelength that are still inside the horizon at η^* are not affected significantly by past history, due to the exponential expansion of the background.

Back-reaction effects are of two types: one is the second-order change in $\delta\langle T_\mu^\nu\rangle = -\langle 0|[\Delta S_3, [\Delta S_3, T_\mu^\nu]]|0\rangle$. The other arises from the first-order interference between ΔS_3 and the cubic terms in v present in T_μ^ν. The worst-case scenario estimate for these terms is [9]

$$\delta\langle T_\mu^\nu\rangle \sim \lambda^2 M^4 + \sqrt{\epsilon}|\lambda|\frac{M^5}{M_{Pl}} \ll O\left(\epsilon\eta' \frac{M_{Pl}^2 H^4}{M^2}\right). \tag{20}$$

These bounds allow for a λ as large as $O(\sqrt{\epsilon\eta}M_{Pl}H^2/M^3)$. This translates into a coefficient for the non-Gaussianity Eq. (19) as large as $O(\sqrt{\epsilon\eta}M_{Pl}H^5/M^3)$. To compare with Maldacena's result [10], we convert his variable ζ into v and we arrive to a non-Gaussianity coefficient $\sqrt{\epsilon}H^4/M_{Pl}$. The ratio of our coefficient to Maldacena's is

$$\frac{\sqrt{\epsilon\eta}M_{Pl}H^5/M^3}{\sqrt{\epsilon}H^4/M_{Pl}} = \sqrt{\eta}\frac{M_{Pl}^2 H}{M^3}. \tag{21}$$

[3] In this section we revert to using conformal time and $a(\eta^*) \neq 1$.

This number can be very large, easily larger than 10^2. Thus, the non-Gaussianity due initial conditions can be observably large, provided, of course, that the initial time η^* is no more than about 60 e-foldings away from the end of inflation. Otherwise, the cutoff at $|k| \sim 1/\eta^*$ would wash out all effects on any observable, sub-horizon fluctuation.

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